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# Nonreciprocal reconfigurable microwave optomechanical circuit

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Nonreciprocal microwave devices are ubiquitous in radar and radio communication and indispensable in the readout chains of superconducting quantum circuits. Since they commonly rely on ferrite materials requiring large magnetic fields that make them bulky and lossy, there has been significant interest in magnetic-field-free on-chip alternatives, such as those recently implemented using the Josephson nonlinearity. Here, we realize reconfigurable nonreciprocal transmission between two microwave modes using purely optomechanical interactions in a superconducting electromechanical circuit. The scheme relies on the interference in two mechanical modes that mediate coupling between the microwave cavities and requires no magnetic field. We analyse the isolation, transmission and the noise properties of this nonreciprocal circuit. Finally, we show how quantum-limited circulators can be realized with the same principle. All-optomechanically mediated nonreciprocity demonstrated here can also be extended to directional amplifiers, and it forms the basis towards realizing topological states of light and sound.

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Nonreciprocal devices, such as isolators, circulators and directional amplifiers, exhibit altered transmission characteristics if the input and output channels are interchanged. They are essential to several applications in signal processing and communication, as they protect devices from interfering signals<sup>1</sup>. At the heart of any such device lies an element breaking Lorentz reciprocal symmetry for electromagnetic sources<sup>2, 3</sup>. Such elements have included ferrite materials<sup>4–6</sup>, magneto-optical materials<sup>7–10</sup>, optical nonlinearities<sup>11–13</sup>, temporal modulation<sup>14–19</sup>, chiral atomic states<sup>20</sup> and physical rotation<sup>21</sup>. Typically, a commercial nonreciprocal microwave apparatus exploits ferrite materials and magnetic fields, which leads to a propagation-direction-dependent phase shift for different field polarizations. A significant drawback of such devices is that they are ill-suited for sensitive superconducting circuits, since their strong magnetic fields are disruptive and require heavy shielding. In recent years, the major advances in quantum superconducting circuits<sup>22</sup>, that require isolation from noise emanating from readout electronics, have led to a significant interest in nonreciprocal devices operating at the microwave frequencies that dispense with magnetic fields and can be integrated on-chip.

As an alternative to ferrite-based nonreciprocal technologies, several approaches have been pursued towards nonreciprocal microwave chip-scale devices. Firstly, the modulation in time of the parametric couplings between modes of a network can simulate rotation about an axis, creating an artificial magnetic field<sup>14, 18, 23, 24</sup> rendering the system nonreciprocal with respect to the ports. Secondly, phase matching of a parametric interaction can lead to nonreciprocity, since the signal only interacts with the pump when copropagating with it and not in the opposite direction. This causes travelling-wave amplification to be directional<sup>24–27</sup>. Phase-matching-induced nonreciprocity can also occur in optomechanical systems<sup>28, 29</sup>, where parity considerations for the interacting spatial modes apply<sup>30–32</sup>. Finally, interference in parametrically coupled multi-mode systems can be used. In these systems, nonreciprocity arises due to interference between multiple coupling pathways along with dissipation in ancillary modes<sup>33</sup>. Here, dissipation is a key resource to break reciprocity, as it forms a flow of energy always leaving the system, even as input and output are interchanged. It has therefore been viewed as reservoir engineering<sup>34</sup>. Following this approach, nonreciprocity has recently been demonstrated in Josephson-junctions-based microwave circuits<sup>35, 36</sup> and in a photonic-crystal-based optomechanical circuit<sup>37</sup>. These realizations and theoretical proposals to achieve nonreciprocity in multi-mode systems rely on a direct, coherent coupling between the electromagnetic input and output modes.

Here, in contrast, we describe a scheme to attain reconfigurable nonreciprocal transmission without a need for any direct coherent coupling between input and output modes, using purely optomechanical interactions<sup>28, 29</sup>. This scheme neither requires cavity–cavity interactions nor phonon–phonon coupling, which are necessary for the recently demonstrated optomechanical nonreciprocity in the optical domain<sup>37</sup>. Two paths of transmission between the microwave modes are established, through two distinct mechanical modes. Interference between those paths with differing phases forms the basis of the nonreciprocal process<sup>38, 39</sup>. In fact, due to the finite quality factor of the intermediary mechanical modes, both conversion paths between the electromagnetic modes are partly dissipative in nature. Nonreciprocity is in this case only possible by breaking the symmetry between the two dissipative coupling pathways. We describe the mechanism in detail below, shedding some light on the essential ingredients for nonreciprocity using this approach.

## Results

**Theoretical model.** We first theoretically model our system to reveal how nonreciprocity arises. We consider two microwave modes (described by their annihilation operators  $\hat{a}_1, \hat{a}_2$ ) having resonance frequencies  $\omega_{c,1}, \omega_{c,2}$  and dissipation rates  $\kappa_1, \kappa_2$ , which are coupled to two mechanical modes (described by the annihilation operators  $\hat{b}_1, \hat{b}_2$ ) having resonance frequencies  $\Omega_1, \Omega_2$  and dissipation rates  $\Gamma_{m,1}, \Gamma_{m,2}$  (Fig. 1a). The radiation-pressure-type optomechanical interaction has the form<sup>28, 29</sup>  $g_{0,ij}\hat{a}_i^\dagger\hat{a}_i(\hat{b}_j + \hat{b}_j^\dagger)$  (in units where  $\hbar = 1$ ), where  $g_{0,ij}$  designates the vacuum optomechanical coupling strength of the  $i^{\text{th}}$  microwave mode to the  $j^{\text{th}}$  mechanical mode. Four microwave tones are applied, close to each of the two lower sidebands of the two microwave modes, with detunings of  $\Delta_{11} = \Delta_{21} = -\Omega_1 - \delta$  and  $\Delta_{12} = \Delta_{22} = -\Omega_2 + \delta$  (Fig. 2c). We linearize the Hamiltonian, neglect counter-rotating terms, and write it in a rotating frame with respect to the mode frequencies (Supplementary Note 1)

$$H = -\delta\hat{b}_1^\dagger\hat{b}_1 + \delta\hat{b}_2^\dagger\hat{b}_2 + g_{11}(\hat{a}_1\hat{b}_1^\dagger + \hat{a}_1^\dagger\hat{b}_1) + g_{21}(\hat{a}_2\hat{b}_1^\dagger + \hat{a}_2^\dagger\hat{b}_1) + g_{12}(\hat{a}_1\hat{b}_2^\dagger + \hat{a}_1^\dagger\hat{b}_2) + g_{22}(e^{i\phi}\hat{a}_2\hat{b}_2^\dagger + e^{-i\phi}\hat{a}_2^\dagger\hat{b}_2) \quad (1)$$

where  $\hat{a}_i$  and  $\hat{b}_j$  are redefined to be the quantum fluctuations around the linearized mean fields. Here  $g_{ij} = g_{0,ij}\sqrt{n_{ij}}$  are the field-enhanced optomechanical coupling strengths, where  $n_{ij}$  is the contribution to the mean intracavity photon number due to the drive with detuning  $\Delta_{ij}$ . Although in principle each coupling is complex, without loss of generality we can take all to be real except the one between  $\hat{a}_2$  and  $\hat{b}_2$  with a complex phase  $\phi$ .

We start by considering frequency conversion through a single mechanical mode. Neglecting the noise terms, the field exiting the cavity  $\hat{a}_2$  is given by  $\hat{a}_{2,\text{out}} = S_{21}\hat{a}_{1,\text{in}} + S_{22}\hat{a}_{2,\text{in}}$ , which defines the scattering matrix  $S_{ij}$ . For a single mechanical pathway, setting  $g_{12} = g_{22} = 0$  and  $\delta = 0$ , the scattering matrix between input and output mode becomes

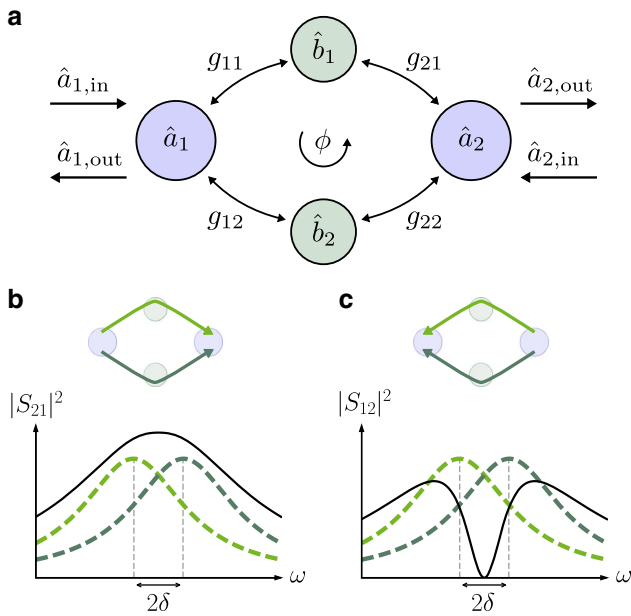
$$S_{21}(\omega) = \sqrt{\frac{\kappa_{\text{ex},1}\kappa_{\text{ex},2}}{\kappa_1\kappa_2}} \frac{\sqrt{\mathcal{C}_{11}\mathcal{C}_{21}}\Gamma_{m,1}}{\frac{\Gamma_{\text{eff},1}}{2} - i\omega}, \quad (2)$$

where  $\kappa_{\text{ex},1}, \kappa_{\text{ex},2}$  denote the external coupling rates of the microwave modes to the feedline, and the (multiphoton) cooperativity for each mode pair is defined as  $\mathcal{C}_{ij} = 4g_{ij}^2/(\kappa_i\Gamma_{m,j})$ . Conversion occurs within the modified mechanical response over an increased bandwidth  $\Gamma_{\text{eff},1} = \Gamma_{m,1}(1 + \mathcal{C}_{11} + \mathcal{C}_{21})$ . This scenario, where a mechanical oscillator mediates frequency conversion between electromagnetic modes, has recently been demonstrated<sup>40</sup> with a microwave optomechanical circuit<sup>41</sup>, and moreover used to create a bidirectional link between a microwave and an optical mode<sup>42</sup>. Optimal conversion, limited by internal losses in the microwave cavities, reaches at resonance  $|S_{21}|_{\text{max}}^2 = \frac{\kappa_{\text{ex},1}\kappa_{\text{ex},2}}{\kappa_1\kappa_2}$  in the limit of large cooperativities  $\mathcal{C}_{11} = \mathcal{C}_{21} \gg 1$ .

We next describe the nonreciprocal transmission of the full system with both mechanical modes. We consider the ratio of transmission amplitudes given by

$$\frac{S_{12}(\omega)}{S_{21}(\omega)} = \frac{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{+i\phi}}{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{-i\phi}} \quad (3)$$

with the mechanical susceptibilities defined as  $\chi_1^{-1}(\omega) = \Gamma_{m,1}/2 - i(\delta + \omega)$  and  $\chi_2^{-1}(\omega) = \Gamma_{m,2}/2 + i(\delta - \omega)$ . Conversion is nonreciprocal if the above expression has a magnitude that differs from 1. If  $S_{21}$  and  $S_{12}$  differ only by a phase, it can be eliminated by a redefinition of either  $\hat{a}_1$  or  $\hat{a}_2$ <sup>24, 33</sup>. Upon a change in conversion direction, the phase  $\phi$  of the coherent coupling (between the microwave and mechanical mode) is



**Fig. 1** Optomechanical nonreciprocal transmission via interference of two asymmetric dissipative coupling pathways. **a** Two microwave modes  $\hat{a}_1$  and  $\hat{a}_2$  are coupled via two mechanical modes  $\hat{b}_1$  and  $\hat{b}_2$  through optomechanical frequency conversion (as given by the coupling constants  $g_{11}, g_{21}, g_{12}, g_{22}$ ). Nonreciprocity is based on the interference between the two optomechanical (conversion) pathways  $g_{11}, g_{21}$  and  $g_{12}, g_{22}$ , in the presence of a suitably chosen phase difference  $\phi$  between the coupling constants as well as the deliberate introduction of an asymmetry in the pathways. **b, c** The symmetry between the pathways can be broken by off-setting the optomechanical transmission windows through each mechanical mode (dashed lines in dark and light green) by a frequency difference  $2\delta$ , resulting in different  $|S_{21}|$  and  $|S_{12}|$  (solid lines). Each single pathway, in the absence of the other mode, is described by Eq. (2). In the forward direction **b**, the two paths interfere constructively, allowing transmission and a finite scattering matrix element  $S_{21}$  on resonance with the first microwave cavity. In contrast, in the backward direction **c**, the paths interfere destructively, such that  $S_{12} \approx 0$ , thereby isolating port 1 from port 2 on resonance with the second microwave cavity. The isolation bandwidth is determined by the intrinsic dissipation rate of the mechanical modes

conjugated, while the complex phase associated with the response of the dissipative mechanical modes remains unchanged. Physically, scattering from  $1 \rightarrow 2$  is related to scattering from  $2 \rightarrow 1$  via time-reversal, which conjugates phases due to coherent evolution of the system. Dissipation is untouched by such an operation and thus remains invariant. Indeed, the mechanical dissipation is an essential ingredient for the nonreciprocity to arise in this system, but not sufficient on its own. In fact, if we align the frequency conversion windows corresponding to the two mechanical modes by setting  $\delta = 0$ , the system becomes reciprocal on resonance ( $\omega = 0$ ), since there is no longer any phase difference between numerator and denominator. This situation corresponds to two symmetric pathways resulting from purely dissipative couplings; they can interfere only in a reciprocal way.

**Conditions for isolation.** We study the conditions for isolation when backward transmission  $S_{12}$  vanishes while forward transmission  $S_{21}$  is non-zero. A finite offset  $2\delta$  between the mechanical conversion windows causes an intrinsic phase shift for a signal on resonance ( $\omega = 0$ ) travelling one path compared to

the other, as it falls either on the red or the blue side of each mechanical resonance. The coupling phase  $\phi$  is then adjusted to cancel propagation in the backward direction  $S_{12}$  (Fig. 1c), by cancelling the two terms in the numerator of Eq. (3). In general, there is always a frequency  $\omega$  for which  $|g_{11}\chi_1(\omega)g_{21}| = |g_{12}\chi_2(\omega)g_{22}|$ , such that the phase  $\phi$  can be tuned to cancel transmission in one direction. Specifically, for two mechanical modes with identical decay rates ( $\Gamma_{m,1} = \Gamma_{m,2} = \Gamma_m$ ) and symmetric couplings ( $g_{11}g_{21} = g_{12}g_{22}$ ), we find that transmission from ports 2 to 1 vanishes on resonance if

$$\frac{\Gamma_m}{2\delta} = \tan \frac{\phi}{2}. \tag{4}$$

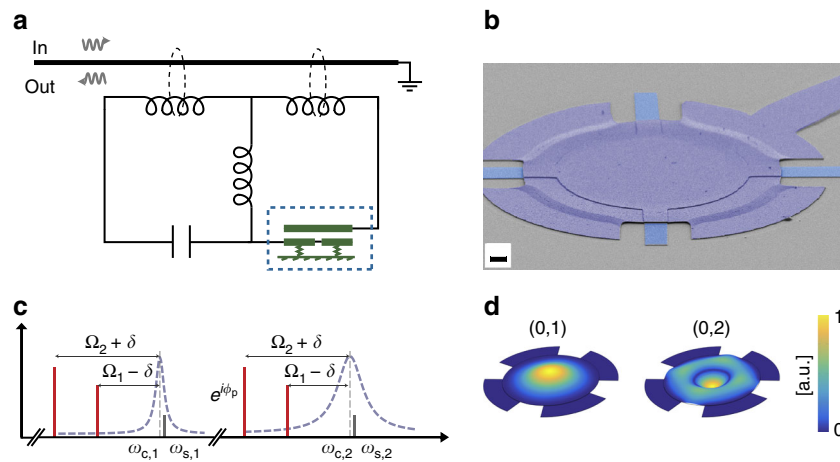
The corresponding terms of the denominator will have a different relative phase, and the signal will add constructively instead, in the forward direction (Fig. 1b). The device thus acts as an isolator from  $\hat{a}_1$  to  $\hat{a}_2$ , realized without relying on the Josephson nonlinearity<sup>35, 36</sup>. We now describe the conditions to minimize insertion loss of the isolator in the forward direction. Still considering the symmetric case, the cooperativity is set to be the same for all modes ( $C_{ij} = C$ ). For a given separation  $\delta$ , transmission on resonance ( $\omega = 0$ ) in the isolating direction has the maximum

$$|S_{21}|_{\max}^2 = \frac{\kappa_{\text{ex},1}\kappa_{\text{ex},2}}{\kappa_1\kappa_2} \left(1 - \frac{1}{2C}\right) \tag{5}$$

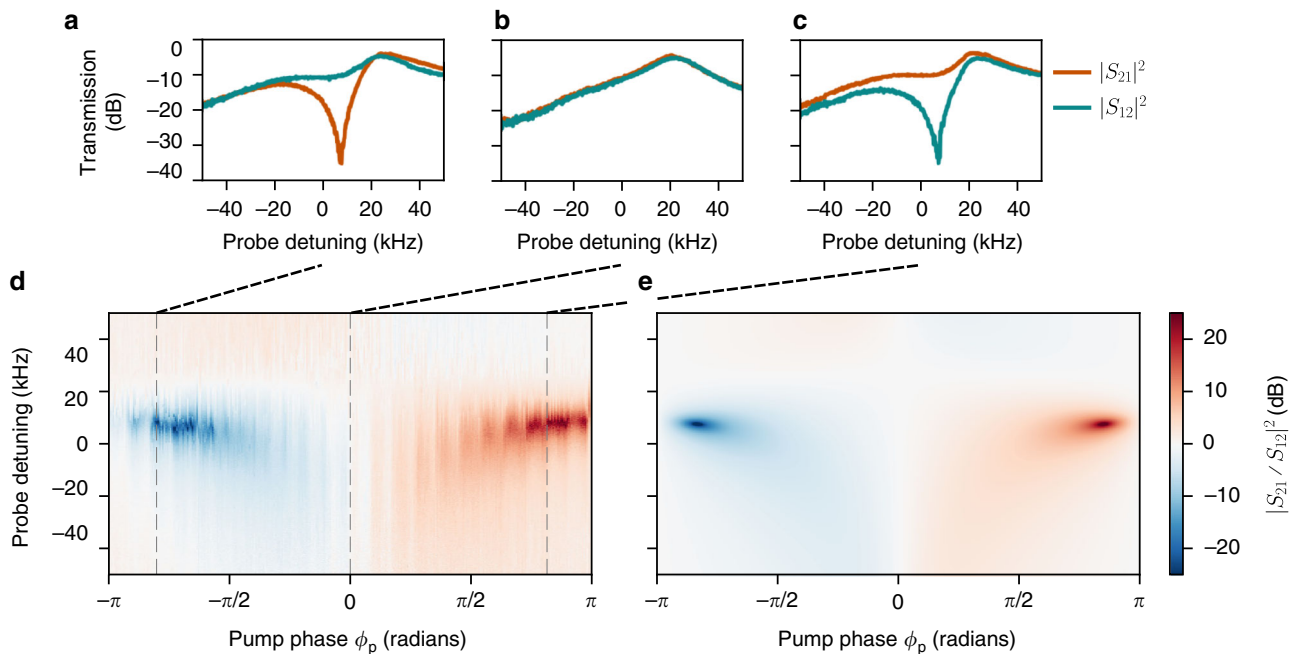
for a cooperativity  $C = 1/2 + 2\delta^2/\Gamma_m^2$ . As in the case for a single mechanical pathway in Eq. (2), for large cooperativity, the isolator can reach an insertion loss only limited by the internal losses of the microwave cavities.

The unusual and essential role of dissipation in this nonreciprocal scheme is also apparent in the analysis of the bandwidth of the isolation. Although the frequency conversion through a single mechanical mode has a bandwidth  $\Gamma_{\text{eff},j}$  (Eq. (2)), caused by the optomechanical damping of the pumps on the lower sidebands, the nonreciprocal bandwidth is set by the intrinsic mechanical damping rates. Examination of Eq. (3) reveals that nonreciprocity originates from the interference of two mechanical susceptibilities of widths  $\Gamma_{m,j}$ . One can conclude that the intrinsic mechanical dissipation, which takes energy out of the system regardless of the transmission direction, is an essential ingredient for the nonreciprocal behaviour reported here, as discussed previously<sup>33, 34</sup>. In contrast, optomechanical damping works symmetrically between input and output modes. By increasing the coupling rates, using higher pump powers, the overall conversion bandwidth increases, while the nonreciprocal bandwidth stays unchanged.

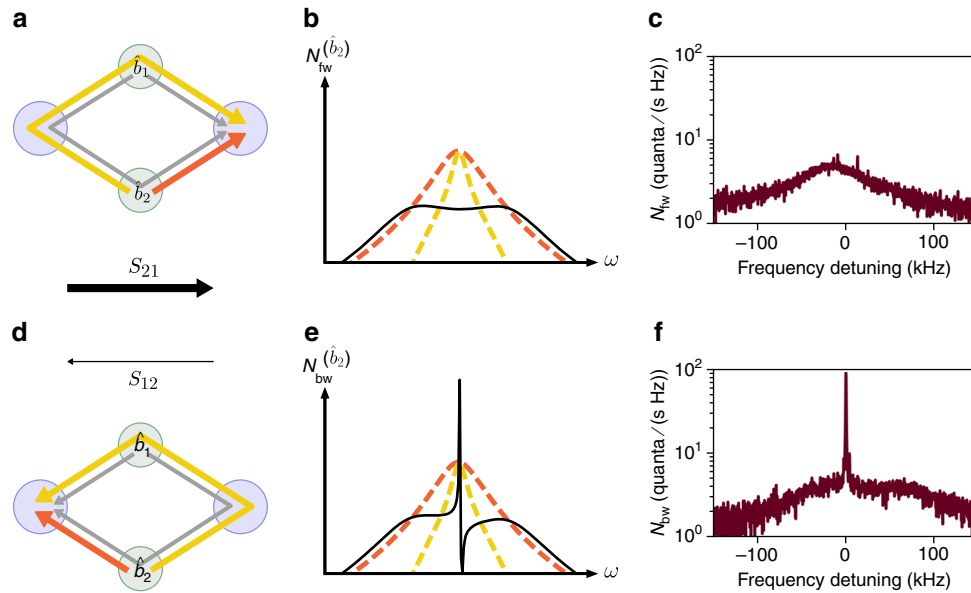
**Experimental realization.** We experimentally realize this nonreciprocal scheme using a superconducting circuit optomechanical system in which mechanical motion is capacitively coupled to a multimode microwave circuit<sup>41</sup>. The circuit, schematically shown in Fig. 2a, supports two electromagnetic modes with resonance frequencies  $(\omega_{c,1}, \omega_{c,2}) = 2\pi \cdot (4.1, 5.2)$  GHz and energy decay rates  $(\kappa_1, \kappa_2) = 2\pi \cdot (0.2, 3.4)$  MHz, both of them coupled to the same vacuum-gap capacitor. We utilize the fundamental and second order radially symmetric (0, 2) modes of the capacitor’s mechanically compliant top plate<sup>43</sup> (Fig. 2b, d) with resonance frequencies  $(\Omega_1, \Omega_2) = 2\pi \cdot (6.5, 10.9)$  MHz, intrinsic energy decay rates  $(\Gamma_{m,1}, \Gamma_{m,2}) = 2\pi \cdot (30, 10)$  Hz and optomechanical vacuum coupling strengths  $(g_{0,11}, g_{0,12}) = 2\pi \cdot (91, 12)$  Hz, respectively (with  $g_{0,11} \approx g_{0,21}$  and  $g_{0,12} \approx g_{0,22}$ , i.e. the two microwave cavities are symmetrically coupled to the mechanical modes). The device is placed at the mixing chamber



**Fig. 2** Implementation of a superconducting microwave circuit optomechanical device for nonreciprocity. **a** A superconducting circuit featuring two electromagnetic modes in the microwave domain is capacitively coupled to a mechanical element (a vacuum-gap capacitor, *dashed rectangle*) and inductively coupled to a microstrip feedline. The end of the feedline is grounded and the circuit is measured in reflection. **b** Scanning electron micrograph of the drum-head-type vacuum gap capacitor (*dashed rectangle* in **a**) with a gap distance below 50 nm, made from aluminium on a sapphire substrate. The scale bar indicates 2  $\mu\text{m}$ . **c** Frequency domain schematic of the microwave pump setup to achieve nonreciprocal mode conversion. Microwave pumps (red bars) are placed at the lower motional sidebands—corresponding to the two mechanical modes—of both microwave resonances (*dashed purple lines*). The pumps are detuned from the exact sideband condition by  $\pm\delta = 2\pi \cdot 18$  kHz, creating two optomechanically induced transparency windows detuned by  $2\delta$  from the microwave resonance frequencies (denoted by  $\omega_{c,1}$  and  $\omega_{c,2}$ , vertical dashed lines). The phase  $\phi_p$  of one of the pumps is tuned. The propagation of an incoming signal (with frequency  $\omega_{s,1}$  or  $\omega_{s,2}$ , solid grey bars) in the forward and backward direction depends on this phase and nonreciprocal microwave transmission can be achieved. **d** Finite-element simulation of the displacement of the fundamental (0, 1) and second order radially symmetric (0, 2) mechanical modes (with measured resonant frequencies  $\Omega_1/2\pi = 6.5$  MHz and  $\Omega_2/2\pi = 10.9$  MHz, respectively) which are exploited as intermediary dissipative modes to achieve nonreciprocal microwave conversion



**Fig. 3** Experimental demonstration of nonreciprocity. **a-c** Power transmission between modes 1 and 2 as a function of probe detuning, shown in both directions for pump phases  $\phi_p = -0.8\pi, 0, 0.8\pi$  radians (respectively **a-c**). Isolation of more than 20 dB in the forward **c** and backward **a** directions is demonstrated, as well as reciprocal behaviour **b**. **d** The ratio of transmission  $|S_{21}/S_{12}|^2$ , representing a measure of nonreciprocity, is shown as a function of pump phase  $\phi_p$  and probe detuning. Two regions of nonreciprocity develop, with isolation in each direction. The system is reconfigurable as the direction of isolation can be swapped by taking  $\phi_p \rightarrow -\phi_p$ . **e** Theoretical ratio of transmission from Eq. (3), calculated with independently estimated experimental parameters. The theoretical model includes effectively lowered cooperativities for the mechanical mode  $\hat{b}_1$  due to cross-damping (optomechanical damping of the lower frequency mechanical mode by the pump on the sideband of the higher frequency mechanical mode) acting as an extra loss channel



**Fig. 4** Asymmetric noise emission of the nonreciprocal circuit. The noise emission is mainly due to mechanical thermal noise, that is converted through two paths to the microwave modes. The resulting interference creates a different noise pattern in the forward **a–c** and the backward **d–f** directions when the circuit is tuned as an isolator from mode  $\hat{a}_1$  to  $\hat{a}_2$ . **a, d** The two possible paths for the noise are shown for each mechanical mode. For  $\hat{b}_2$ , the direct path (orange) and the indirect path going through mode  $\hat{b}_1$  (yellow) are highlighted (the corresponding paths for  $\hat{b}_1$  are shown in grey). **b, e** Each path on its own would result in a wide noise spectrum that is equally divided between the two microwave cavities (dashed yellow and orange lines). When both paths are available, however, the noise interferes differently in each direction (solid lines). In the backward direction **e**, a sharp interference peak appears, of much larger amplitude than the broad base. The theoretical curves (on an arbitrary logarithmic scale) are shown for the symmetric case ( $\Gamma_{m,1} = \Gamma_{m,2}$ ) and for the single mode  $\hat{b}_2$ . Note that for the mode  $\hat{b}_1$ , the shape of the asymmetric peak in the backward noise would be the mirror image. **c, f** Measured output spectra of modes  $\hat{a}_2$  (**c**) and  $\hat{a}_1$  (**f**), calibrated to show the photon flux leaving the circuit. Because cross-damping provides extra cooling for the mode  $\hat{b}_1$ , the thermal noise of  $\hat{b}_2$  is expected to dominate

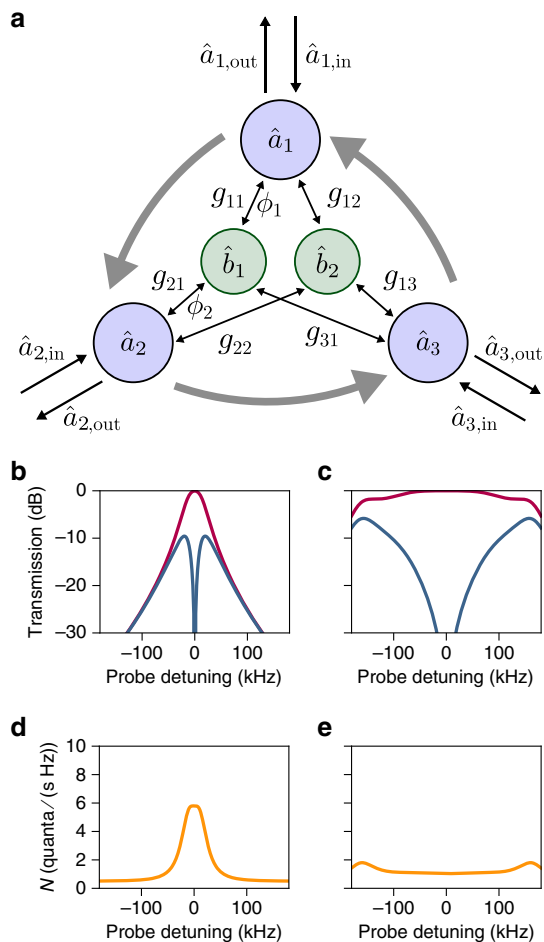
of a dilution refrigerator at 200 mK and all four incoming pump tones are heavily filtered and attenuated to eliminate Johnson and phase noise (details are published elsewhere<sup>44</sup>). We establish a parametric coupling between the two electromagnetic and the two mechanical modes by introducing four microwave pumps with frequencies slightly detuned from the lower motional sidebands of the resonances, as shown in Fig. 2c and as discussed above. An injected probe signal  $\omega_{s1(s2)}$  around the lower (higher) frequency microwave mode is then measured in reflection using a vector network analyser.

Frequency conversion in both directions,  $|S_{21}(\omega)|^2$  and  $|S_{12}(\omega)|^2$ , are measured and compared in Fig. 3a–c. The powers of the four pumps are chosen such that the associated individual cooperativities are given by  $C_{11} = 520$ ,  $C_{21} = 450$ ,  $C_{12} = 1350$  and  $C_{22} = 1280$ . The detuning from the lower motional sidebands is set to  $\delta = 2\pi \cdot 18$  kHz. By pumping both cavities on the lower sideband associated with the same mechanical mode, a signal injected on resonance with one of the modes will be frequency converted to the other mode. This process can add negligible noise, when operating with sufficiently high cooperativity, as demonstrated recently<sup>40</sup>. In the experiment, the four drive tones are all phase-locked and the phase of one tone  $\phi_p$  is varied continuously from  $-\pi$  to  $\pi$ . The pump phase is linked to the coupling phase  $\phi$  by a constant offset, in our case  $\phi_p \approx \phi + \pi$ . Between the two transmission peaks corresponding to each mechanical mode, a region of nonreciprocity develops, depending on the relative phase  $\phi_p$ .

The amount of reciprocity that occurs in this process is quantified and measured by the ratio of forward to backward conversion  $|S_{21}/S_{12}|^2$ . Figure 3d shows this quantity as a function of probe detuning and the relative pump phase. Isolation of more than 20 dB is demonstrated in each direction in a reconfigurable manner, i.e. the direction of isolation can be

switched by taking  $\phi_p \rightarrow -\phi_p$ , as expected from Eq. (4). The ideal theoretical model, which takes into account  $\Gamma_{m,1} \neq \Gamma_{m,2}$ , predicts that the bandwidth of the region of nonreciprocity is commensurate with the arithmetic average of the bare mechanical dissipation rates,  $\sim 2\pi \cdot 20$  Hz. However, given the significantly larger coupling strength of the fundamental mechanical mode compared to the second order mode, and that  $\kappa_2/\Omega_{1,2}$  is not negligible, the pump detuned by  $\Omega_2 - \delta$  from the microwave mode  $\hat{a}_2$  introduces considerable cross-damping (i.e. resolved sideband cooling) for the fundamental mode. This cross-damping, measured separately to be  $\Gamma_{m,1}^{(\text{cross})} \approx 2\pi \cdot 20$  kHz at the relevant pump powers, widens the bandwidth of nonreciprocal behaviour by over two orders of magnitude and effectively cools the mechanical oscillator. It also acts as a loss in the frequency conversion process and therefore effectively lowers the cooperativities to  $(C_{11}, C_{21}) \approx (0.78, 0.68)$ . This lowered cooperativity accounts for the overall  $\sim 10$  dB loss in the forward direction. This limitation can be overcome in a future design by increasing the sideband resolution with decreased  $\kappa_i$  or utilising the fundamental modes of two distinct mechanical elements with similar coupling strengths. To compare the experiment to the theory we use a model that takes into account the cross-damping and an increased effective mechanical dissipation of the fundamental mode. The model is compared to the experimental data in Fig. 3e, showing good qualitative agreement.

**Noise properties.** From a technological standpoint, it does not suffice for an isolator to have the required transmission properties; since its purpose is to protect the input from any noise propagating in the backward direction, the isolator’s own noise emission is relevant. We, therefore, return to the theoretical description of the ideal symmetric case and derive the noise



**Fig. 5** Proposal for a microwave optomechanical circulator. **a** With a third microwave mode  $\hat{a}_3$  coupled to the same two mechanical oscillators, circulation can be achieved between the three microwave cavities. The circuit now involves two independent loops, with two phases  $\phi_1$  and  $\phi_2$  that can be tuned with the phases associated with  $g_{21}$  and  $g_{11}$ , respectively. **b, c** The theoretical transmission in the circulating direction (counter-clockwise, in red) and the opposite direction (clockwise, in blue) are shown for the cooperativities  $C = 100$  (**b**) and  $C = 1000$  (**c**). The isolation bandwidth scales with  $C$  and is only limited by the energy decay rates of the microwave modes. Experimentally realistic parameters are chosen with overcoupled cavities of energy decay rates  $\kappa_1 = \kappa_2 = \kappa_3 = 2\pi \cdot 200$  kHz and  $\Gamma_{m,1} = \Gamma_{m,2} = 2\pi \cdot 100$  Hz. **d, e** Noise emission spectra for the same two cooperativities ( $C = 100$  (**d**) and  $C = 1000$  (**e**)), for  $\bar{n}_{m,1} = \bar{n}_{m,2} = 800$ . Note that for the circulator the noise is symmetric for all the cavities, and that it decreases with increasing cooperativity

properties expected from the device, in the limit of overcoupled cavities ( $\kappa_{ex,j} \approx \kappa_j$ ). In the forward direction and on resonance, the emitted noise amounts to  $N_{fw}(0) = 1/2 + (\bar{n}_{m,1} + \bar{n}_{m,2})/(4C)$ , where  $\bar{n}_{m,j}$  is the thermal occupation of each mechanical mode (Supplementary Note 2). In the limit of low insertion loss and large cooperativity, the added noise becomes negligible in the forward direction. More relevant for the purpose of using an isolator to protect sensitive quantum apparatus is the noise emitted in the backward direction, given by  $N_{bw}(0) = 1/2 + (\bar{n}_{m,1} + \bar{n}_{m,2})/2$ . Here the noise is directly commensurate with the occupation of the mechanics which can be of hundreds of quanta even at cryogenic millikelvin temperatures, due to the low mechanical frequencies. This is a direct consequence of isolation without reflection, since it prevents fluctuations from either cavity to emerge in the

backward direction. In order to preserve the commutation relations of the bosonic output modes, the fluctuations consequently have to originate from the mechanical modes. A practical low-noise design, therefore, requires a scheme to externally cool the mechanical modes, e.g. via sideband cooling using an additional auxiliary microwave mode.

The origin of this noise asymmetry can be understood as noise interference. The thermal fluctuations of one mechanical oscillator are converted to microwave noise in each cavity through two paths, illustrated in Fig. 4a, d: a direct (orange) and an indirect (yellow) link. Each pathway, on its own and with the same coupling strength, would result in symmetric noise that decreases in magnitude with increasing cooperativity. When both are present, however, the noise interferes with itself differently in each direction (Supplementary Note 3). In the forward direction, the noise interferes destructively (Fig. 4b) leading to low added noise, but in the backward direction, a sharp interference peak arises (Fig. 4e) with finite noise in the nonreciprocal bandwidth even in the high-cooperativity limit. In an intuitive picture, the circuit acts as a circulator that routes noise from the output port to the mechanical thermal bath and in turn the mechanical noise to the input port. We demonstrate experimentally the noise asymmetry by detecting the output spectra at each microwave mode while the device isolates the mode  $\hat{a}_1$  from  $\hat{a}_2$  by more than 25 dB (Fig. 4c, f). The cooperativities are here set to  $(C_{11}, C_{21}, C_{12}, C_{22}) = (20.0, 14.2, 106, 89)$  with a cross-damping  $\Gamma_{m,1}^{(cross)} \approx 2\pi \cdot 2.6$  kHz, in order to optimize the circuit for a lower insertion loss and increase the noise visibility. As there is additional cooling from the off-resonant pump on mode  $\hat{b}_1$ , we expect noise from  $\hat{b}_2$  to dominate.

**Quantum-limited circulator.** There exists a way to circumvent the mechanical noise entirely: introducing one extra microwave mode  $\hat{a}_3$ , we can realize a circulator, where instead of mechanical fluctuations, the fluctuations from the third microwave mode emerge in the backward direction. The scheme is illustrated in Fig. 5a. As before, the two mechanical modes are used to create two interfering pathways, now between the three microwave cavities. Since there are now two independent loops, two phases matter; we choose the phases associated to the couplings  $g_{11}$  and  $g_{21}$  and set them respectively to  $\phi_1 = 2\pi/3$  and  $\phi_2 = -2\pi/3$ . With the mechanical detunings set to  $\delta_i = \frac{\sqrt{3}}{2}(C + \frac{1}{3})\Gamma_{m,i}$ , the system then becomes a circulator that routes the input of port  $\hat{a}_1$  to  $\hat{a}_2$ ,  $\hat{a}_2$  to  $\hat{a}_3$  and so on (Supplementary Note 4). Critically and in contrast to above, counter-propagating signals are not dissipated in the mechanical oscillators, but directed to the other port, with two advantages. First, the bandwidth of nonreciprocity is not limited to the mechanical dissipation rate but instead increases with  $C$  until reaching the ultimate limit given by the cavity linewidth (Fig. 5b, c). Second, the mechanical noise emission is symmetrically spread between the three modes, and over the wide conversion bandwidth (Fig. 5d, e). In the large cooperativity limit, the nonreciprocal process becomes quantum limited, irrespective of the temperature of the mechanical thermal baths.

## Discussion

In conclusion, we described and experimentally demonstrated a new scheme for reconfigurable nonreciprocal transmission in the microwave domain using a superconducting optomechanical circuit. This scheme is based purely on optomechanical couplings, thus it alleviates the need for coherent microwave cavity-cavity (or direct phonon-phonon) interactions, and significantly facilitates the experimental realization, in contrast to recently

used approaches of optomechanical nonreciprocity in the optical domain<sup>37</sup>. Nonreciprocity arises due to interference in the two mechanical modes, which mediate the microwave cavity–cavity coupling. This interference also manifests itself in the asymmetric noise output of the circuit. This scheme can be readily extended to implement quantum-limited phase-preserving and phase-sensitive directional amplifiers<sup>45</sup>. Moreover, an additional microwave mode enables quantum-limited microwave circulators on-chip with large bandwidth, limited only by the energy decay rate of the microwave modes. Finally, the presented scheme can be generalized to an array, and thus can form the basis to create topological phases of light and sound<sup>46</sup> or topologically protected chiral amplifying states<sup>47</sup> in arrays of electromechanical circuits, without requiring cavity–cavity or phonon–phonon mode hopping interactions.

**Data availability.** The code and data used to produce the plots within this paper are available at <http://dx.doi.org/10.5281/zenodo.816171>. All other data used in this study are available from the corresponding authors upon reasonable request.

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## Author contributions

T.J.K. and A.K.F. initiated and directed the project. The nonreciprocity scheme was conceived by D.T., N.B., A.K.F., and T.J.K., A.K., N.R.B. and A.K.F. derived the theoretical model for the isolator. D.M. derived the theoretical noise description. M.A.I. derived the theoretical model for the circulator. A.N. contributed to the theoretical framework. L.D.T. fabricated the devices. N.R.B. and L.D.T. under the supervision of A.

K.F. performed the measurements. N.R.B. and L.D.T. wrote the manuscript with significant contributions from all authors.

### Additional information

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### **Description of Supplementary Files**

File Name: Supplementary Information

Description: Supplementary Figure, Supplementary Notes and Supplementary References

File Name: Peer Review File

## SUPPLEMENTARY NOTE 1: THEORETICAL BACKGROUND

In this note we derive the effective Hamiltonian relevant for our system, calculate the input-output scattering matrix for the electromagnetic modes and discuss the conditions for obtaining nonreciprocal microwave transmission.

We consider two mechanical degrees of freedom whose positions parametrically modulate the frequencies of two electromagnetic modes via radiation-pressure coupling [1]. The Hamiltonian describing this situation is given by ( $\hbar = 1$ )

$$\hat{H} = \sum_{i=1}^2 \left( \omega_{c,i} \hat{a}_i^\dagger \hat{a}_i + \Omega_i \hat{b}_i^\dagger \hat{b}_i \right) + \hat{H}_{\text{int}} + \hat{H}_{\text{drive}}, \quad (1)$$

where  $\hat{a}_1$  and  $\hat{a}_2$  are the annihilation operators associated respectively with the two electromagnetic modes with frequencies  $\omega_{c,1}$  and  $\omega_{c,2}$ ,  $\hat{b}_1$  and  $\hat{b}_2$  are those for the two mechanical modes with respective mechanical frequencies  $\Omega_1$  and  $\Omega_2$ , and  $\hat{H}_{\text{drive}}$  describes the electromagnetic pumps. Radiation-pressure coupling between the microwave and mechanical modes is described by the interaction Hamiltonian [1]

$$\hat{H}_{\text{int}} = - \sum_{j=1}^2 \sum_{k=1}^2 g_{0,jk} \hat{a}_j^\dagger \hat{a}_j (\hat{b}_k + \hat{b}_k^\dagger), \quad (2)$$

with  $g_{0,jk}$  the vacuum optomechanical coupling strength between electromagnetic mode  $j$  and mechanical mode  $k$  and where we neglect cross coupling terms  $\propto \hat{a}_i^\dagger \hat{a}_j$ , which is a good approximation for spectrally distinct modes  $|\omega_{c,1} - \omega_{c,2}| \gg \Omega_i$  [2, 3].

In the experiment, both cavity modes are driven with two microwave tones each. These four tones are close to the lower mechanical sidebands, but the ones driving the mechanical sidebands at frequency  $\Omega_1$  are slightly detuned to the red, whereas the ones driving the sidebands at frequency  $\Omega_2$  are slightly detuned to the blue from the lower sideband. That is, the detuning of the four drives are  $\Delta_{jk} = \omega_{jk} - \omega_{c,j}$  with  $\Delta_{11} = \Delta_{21} = -\Omega_1 - \delta$  and  $\Delta_{12} = \Delta_{22} = -\Omega_2 + \delta$ .

We separate mean and fluctuations in the microwave fields and move to a frame rotating at the cavity frequencies

$$\hat{a}_j = e^{-i\omega_{c,j}t} \left( (\delta \hat{a}_j) + \sum_{k=1}^2 \alpha_{jk} e^{-i\Delta_{jk}t} \right) \quad (3)$$

where  $\alpha_{jk}$  is the coherent state amplitude due to the microwave drive with detuning  $\Delta_{jk}$  with  $j, k = 1, 2$  and  $(\delta \hat{a}_j)$  describe the fluctuations of the two microwave modes  $j = 1, 2$ . We then linearize the Hamiltonian by approximating

$$\hat{a}_j^\dagger \hat{a}_j \approx (\delta \hat{a}_j^\dagger) \left( \sum_{k=1}^2 \alpha_{jk} e^{-i\Delta_{jk}t} \right) + \text{H.c.} \quad (4)$$

To obtain a time-independent Hamiltonian we will assume that the system is in the resolved-sideband limit with respect to both mechanical modes, i.e.  $\Omega_1, \Omega_2 \gg \kappa_1, \kappa_2$ , and that the two mechanical modes are well separated in frequency, i.e.  $|\Omega_1 - \Omega_2| \gg \Gamma_{m,1}, \Gamma_{m,2}$ . Moving into a rotating frame with respect to the free evolution of the microwave modes, and keeping only non-rotating terms, we obtain the effective Hamiltonian describing our system, which is given as equation (1) in the main manuscript,

$$\hat{H} = -\delta \hat{b}_1^\dagger \hat{b}_1 + \delta \hat{b}_2^\dagger \hat{b}_2 + g_{11} (\hat{a}_1 \hat{b}_1^\dagger + \hat{a}_1^\dagger \hat{b}_1) + g_{21} (\hat{a}_2 \hat{b}_1^\dagger + \hat{a}_2^\dagger \hat{b}_1) + g_{12} (\hat{a}_1 \hat{b}_2^\dagger + \hat{a}_1^\dagger \hat{b}_2) + g_{22} (e^{i\phi} \hat{a}_2 \hat{b}_2^\dagger + e^{-i\phi} \hat{a}_2^\dagger \hat{b}_2). \quad (5)$$

Here,  $g_{jk} = g_{0,jk} |\alpha_{jk}|$  are the optomechanical coupling strengths enhanced by the mean intracavity photon numbers  $n_{jk} = |\alpha_{jk}|^2$  due to the drive at frequency  $\omega_{jk}$  and where we have renamed  $(\delta \hat{a}_j) \rightarrow \hat{a}_j$  for notational convenience. Without loss of generality, the phase of all but one coupling constant  $g_{jk}$  can be chosen real. Here, we take all of them real and write out the phase  $\phi$  explicitly which is varied in our experiment.

From the Hamiltonian (supplementary eq. (5)) we derive the equations of motion for our system which can be written in matrix form as [4, 5]

$$\dot{\mathbf{u}} = M \mathbf{u} + L \mathbf{u}_{\text{in}} \quad (6)$$

with  $\mathbf{u} = (\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)^T$ ,  $\mathbf{u}_{\text{in}} = (\hat{a}_{1,\text{in}}, \hat{a}_{2,\text{in}}, \hat{a}_{1,\text{in}}^{(0)}, \hat{a}_{2,\text{in}}^{(0)}, \hat{b}_{1,\text{in}}, \hat{b}_{2,\text{in}})^T$  and  $\mathbf{u}_{\text{out}} = (\hat{a}_{1,\text{out}}, \hat{a}_{2,\text{out}}, \hat{a}_{1,\text{out}}^{(0)}, \hat{a}_{2,\text{out}}^{(0)}, \hat{b}_{1,\text{out}}, \hat{b}_{2,\text{out}})^T$ , where  $\hat{a}_{i,\text{in/out}}$  are the input-output modes of the external microwave feedline and  $\hat{a}_{i,\text{in/out}}^{(0)}$  are those corresponding to internal dissipation.

The matrix  $M$  reads

$$M = \begin{pmatrix} -\frac{\kappa_1}{2} & 0 & -ig_{11} & -ig_{12} \\ 0 & -\frac{\kappa_2}{2} & -ig_{21} & -ig_{22}e^{-i\phi} \\ -ig_{11} & -ig_{21} & +i\delta - \frac{\Gamma_{m,1}}{2} & 0 \\ -ig_{12} & -ig_{22}e^{+i\phi} & 0 & -i\delta - \frac{\Gamma_{m,2}}{2} \end{pmatrix} \quad (7)$$

where the cavity dissipation rates are the sum of external and internal dissipation rates, i.e.  $\kappa_1 = \kappa_{\text{ex},1} + \kappa_{0,1}$  and  $\kappa_2 = \kappa_{\text{ex},2} + \kappa_{0,2}$ , and the matrix  $L$  reads

$$L = \begin{pmatrix} \sqrt{\kappa_{\text{ex},1}} & 0 & \sqrt{\kappa_{0,1}} & 0 & 0 & 0 \\ 0 & \sqrt{\kappa_{\text{ex},2}} & 0 & \sqrt{\kappa_{0,2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\Gamma_{\text{m},1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\Gamma_{\text{m},2}} \end{pmatrix}. \quad (8)$$

Using the input-output relations for a one-sided cavity [4, 5]

$$\mathbf{u}_{\text{out}} = \mathbf{u}_{\text{in}} - L^T \mathbf{u} \quad (9)$$

we can solve the input-output problem in the Fourier domain

$$\mathbf{u}_{\text{out}}(\omega) = S(\omega) \mathbf{u}_{\text{in}}(\omega) \quad (10)$$

with the scattering matrix

$$S(\omega) = \mathbb{1}_{6 \times 6} + L^T [ +i\omega \mathbb{1}_{4 \times 4} + M ]^{-1} L. \quad (11)$$

Eliminating the mechanical degrees of freedom from the equations of motion (supplementary eq. (6)) we obtain

$$\begin{pmatrix} \frac{\kappa_1}{2} - i\omega + g_{11}^2 \chi_1(\omega) + g_{12}^2 \chi_2(\omega) & g_{11} \chi_1(\omega) g_{21} + g_{12} \chi_2(\omega) g_{22} e^{+i\phi} \\ g_{11} \chi_1(\omega) g_{21} + g_{12} \chi_2(\omega) g_{22} e^{-i\phi} & \frac{\kappa_2}{2} - i\omega + g_{21}^2 \chi_1(\omega) + g_{22}^2 \chi_2(\omega) \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} \\ = \begin{pmatrix} \sqrt{\kappa_{\text{ex},1}} \hat{a}_{\text{in},1} + \sqrt{\kappa_{0,1}} \hat{a}_{\text{in},1}^{(0)} - ig_{11} \chi_1(\omega) \sqrt{\Gamma_{\text{m},1}} \hat{b}_{1,\text{in}} - ig_{12} \chi_2(\omega) \sqrt{\Gamma_{\text{m},2}} \hat{b}_{2,\text{in}} \\ \sqrt{\kappa_{\text{ex},2}} \hat{a}_{\text{in},2} + \sqrt{\kappa_{0,2}} \hat{a}_{\text{in},2}^{(0)} - ig_{21} \chi_1(\omega) \sqrt{\Gamma_{\text{m},1}} \hat{b}_{1,\text{in}} - ig_{22} \chi_2(\omega) e^{-i\phi} \sqrt{\Gamma_{\text{m},2}} \hat{b}_{2,\text{in}} \end{pmatrix} \quad (12)$$

where we introduced the mechanical susceptibilities  $\chi_1^{-1}(\omega) = \Gamma_{\text{m},1}/2 - i(\delta + \omega)$  and  $\chi_2^{-1}(\omega) = \Gamma_{\text{m},2}/2 + i(\delta - \omega)$ . Inverting the matrix in supplementary eq. (12) and exploiting the input-output relation (supplementary eq. (9)), we obtain equation (3) of the main manuscript

$$\frac{S_{12}(\omega)}{S_{21}(\omega)} = \frac{g_{11} \chi_1(\omega) g_{21} + g_{12} \chi_2(\omega) g_{22} e^{+i\phi}}{g_{11} \chi_1(\omega) g_{21} + g_{12} \chi_2(\omega) g_{22} e^{-i\phi}}. \quad (13)$$

Note that the expressions in the nominator and denominator in supplementary eq. (13) are equal to the matrix elements coupling the two electromagnetic modes in supplementary eq. (12) which are the sum of the two (complex) amplitudes for the two dissipative optomechanical pathways. Supplementary eq. (13) is used to generate Fig. 3 E of the main text, with all the parameters ( $\Gamma_{\text{m},j}, g_{ij}$ ) independently measured.

For identical mechanical decay rates  $\Gamma_{\text{m},1} = \Gamma_{\text{m},2} = \Gamma_{\text{m}}$  and identical cooperativities  $\mathcal{C} = \mathcal{C}_{ij} = \frac{4g_{ij}^2}{\kappa_i \Gamma_{\text{m},j}}$  we find that the transmission  $2 \rightarrow 1$  vanishes on resonance  $\omega = 0$ , i.e.  $S_{12} = 0$ , if

$$\frac{\Gamma_{\text{m}}}{2\delta} = \tan \frac{\phi}{2}. \quad (14)$$

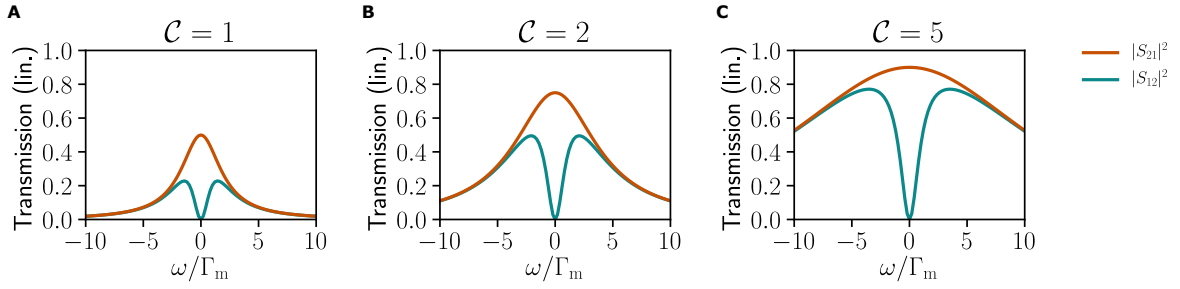
For a given  $\delta$ , maximal transmission in the opposite direction  $1 \rightarrow 2$  is then obtained for  $\mathcal{C} = \frac{1}{2} + \frac{2\delta^2}{\Gamma_{\text{m}}^2}$  and given by

$$|S_{21}|_{\text{max}}^2 = \frac{\kappa_{\text{ex},1} \kappa_{\text{ex},2}}{\kappa_1 \kappa_2} \frac{4\delta^2}{\Gamma_{\text{m}}^2 + 4\delta^2} = \frac{\kappa_{\text{ex},1} \kappa_{\text{ex},2}}{\kappa_1 \kappa_2} \left( 1 - \frac{1}{2\mathcal{C}} \right). \quad (15)$$

We see that for  $\delta \gg \Gamma_{\text{m}}$  the optimal cooperativity  $\mathcal{C} \rightarrow \infty$  and  $|S_{21}(0)|^2 \rightarrow 1$ . Thus, we see that in this limit the electromagnetic scattering matrix of our system becomes that of an ideal isolator, i.e.  $S_{11} = S_{12} = S_{22} = 0$  and  $|S_{21}| = 1$ .

The full scattering matrix  $S_{ij}$  of supplementary eq. (11) is used in Supplementary Fig. 1 to show optimal transmission in each direction for the symmetric case, with different values of the cooperativity. As the cooperativity increases, the overall bandwidth of conversion increases to  $\Gamma_{\text{eff}}$ , but the nonreciprocal bandwidth stays constant. This can be seen in the ratio  $S_{12}(\omega)/S_{21}(\omega)$  in supplementary eq. (13) that depends only on the bare mechanical susceptibilities  $\chi_1(\omega)$  and  $\chi_2(\omega)$ .

For unequal decay rates  $\Gamma_{\text{m},1} \neq \Gamma_{\text{m},2}$ , but equal effective decay rates of the mechanical modes  $\Gamma_{\text{eff},j} = \Gamma_{\text{m},j}(1 + \mathcal{C}_{1j} + \mathcal{C}_{2j})$ , nonreciprocity is obtained for  $\frac{\Gamma_{\pm}}{2\delta} = \tan \frac{\phi}{2}$  off-resonance at a frequency  $\omega = \frac{\Gamma_{+}\Gamma_{-}}{4\delta}$  where  $\Gamma_{\pm} = \frac{1}{2}(\Gamma_{\text{m},1} \pm \Gamma_{\text{m},2})$ . For unequal decay rates  $\Gamma_{\text{m},1} \neq \Gamma_{\text{m},2}$ , but matched cooperativities  $\mathcal{C}_{jk} = \mathcal{C}$ , we find nonreciprocity for  $\frac{\Gamma_{\pm}}{2\delta} = \tan \frac{\phi}{2}$ , but at  $\omega = -\frac{\Gamma_{-}\delta}{\Gamma_{+}}$ .



Supplementary Fig. 1. Microwave transmission of the nonreciprocal electromechanical device in each direction for different values of the cooperativity  $\mathcal{C}$ , derived from supplementary eq. (11) for the case of symmetric mechanical modes ( $\Gamma_{m,1} = \Gamma_{m,2} = \Gamma_m$ ). The detuning  $\delta$  and the phase  $\phi$  are set for maximal transmission according to supplementary eq. (15). As the cooperativity is increased, the overall bandwidth of the frequency conversion increases to  $\Gamma_{\text{eff}}$ , however the bandwidth of nonreciprocal transmission stays constant and is on the order of the intrinsic mechanical damping rate  $\Gamma_m$ . This illustrates the fact that the intrinsic dissipation of the mechanical oscillator is the underlying resource for the nonreciprocity.

### SUPPLEMENTARY NOTE 2: NOISE ANALYSIS OF THE DEVICE

In this note we analyse the noise properties of the nonreciprocal electromechanical device. We assume the bosonic input noise operators obey

$$\langle \hat{a}_{1,\text{in}}(t) \hat{a}_{1,\text{in}}^\dagger(t') \rangle = \delta(t - t') \quad (16)$$

$$\langle \hat{a}_{2,\text{in}}(t) \hat{a}_{2,\text{in}}^\dagger(t') \rangle = \delta(t - t') \quad (17)$$

$$\langle \hat{a}_{1,\text{in}}^{(0)}(t) \hat{a}_{1,\text{in}}^{(0)\dagger}(t') \rangle = \delta(t - t') \quad (18)$$

$$\langle \hat{a}_{2,\text{in}}^{(0)}(t) \hat{a}_{2,\text{in}}^{(0)\dagger}(t') \rangle = \delta(t - t') \quad (19)$$

$$\langle \hat{b}_{1,\text{in}}(t) \hat{b}_{1,\text{in}}^\dagger(t') \rangle = (\bar{n}_{m,1} + 1) \delta(t - t') \quad (20)$$

$$\langle \hat{b}_{2,\text{in}}(t) \hat{b}_{2,\text{in}}^\dagger(t') \rangle = (\bar{n}_{m,2} + 1) \delta(t - t'), \quad (21)$$

i.e. the baths of the microwave modes are assumed to be at zero temperature whereas the mechanical modes have a finite thermal occupation  $\bar{n}_{m,1}$  and  $\bar{n}_{m,2}$ , respectively.

The symmetrised output noise spectra [5] are determined by the scattering matrix of the device (supplementary eq. (11)) as well as the noise properties of the microwave and mechanical baths (supplementary eq. (16) to (21)). Explicitly, we find that the cavity output spectra are given by

$$\begin{aligned} \bar{S}_{1,\text{out}}(\omega) &= \frac{1}{2} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \hat{a}_{1,\text{out}}^\dagger(t) \hat{a}_{1,\text{out}}(0) + \hat{a}_{1,\text{out}}(0) \hat{a}_{1,\text{out}}^\dagger(t) \rangle \\ &= \frac{1}{2} [ |S_{11}(-\omega)|^2 + |S_{12}(-\omega)|^2 + |S_{13}(-\omega)|^2 + |S_{14}(-\omega)|^2 ] + |S_{15}(-\omega)|^2 (\bar{n}_{m,1} + \frac{1}{2}) + |S_{16}(-\omega)|^2 (\bar{n}_{m,2} + \frac{1}{2}) \end{aligned} \quad (22)$$

and

$$\begin{aligned} \bar{S}_{2,\text{out}}(\omega) &= \frac{1}{2} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \hat{a}_{2,\text{out}}^\dagger(t) \hat{a}_{2,\text{out}}(0) + \hat{a}_{2,\text{out}}(0) \hat{a}_{2,\text{out}}^\dagger(t) \rangle \\ &= \frac{1}{2} [ |S_{21}(-\omega)|^2 + |S_{22}(-\omega)|^2 + |S_{23}(-\omega)|^2 + |S_{24}(-\omega)|^2 ] + |S_{25}(-\omega)|^2 (\bar{n}_{m,1} + \frac{1}{2}) + |S_{26}(-\omega)|^2 (\bar{n}_{m,2} + \frac{1}{2}). \end{aligned} \quad (23)$$

In the limit of overcoupled cavities  $\kappa_{\text{ex},i} \approx \kappa_i$  and for the optimal phase  $\phi$  and detuning  $\delta$ , the noise emitted in the backward direction  $2 \rightarrow 1$  on resonance  $\omega = 0$  is

$$\begin{aligned} N_{\text{bw}} = \bar{S}_{1,\text{out}}(0) &= |S_{11}|^2 \times \frac{1}{2} + |S_{12}|^2 \times \frac{1}{2} + |S_{15}|^2 \times \left( \bar{n}_{m,1} + \frac{1}{2} \right) + |S_{16}|^2 \times \left( \bar{n}_{m,2} + \frac{1}{2} \right) \\ &= 0 \times \frac{1}{2} + 0 \times \frac{1}{2} + \frac{1}{2} \times \left( \bar{n}_{m,1} + \frac{1}{2} \right) + \frac{1}{2} \times \left( \bar{n}_{m,2} + \frac{1}{2} \right) \\ &= \frac{1}{2} + \frac{\bar{n}_{m,1} + \bar{n}_{m,2}}{2}, \end{aligned} \quad (24)$$

i.e. in the backward direction the noise of the device is dominated by the noise emitted by the mechanical oscillators.

The noise emitted in the forward direction  $1 \rightarrow 2$  on resonance  $\omega = 0$  is

$$\begin{aligned} N_{\text{fw}} = \bar{S}_{2,\text{out}}(0) &= |S_{21}|^2 \times \frac{1}{2} + |S_{22}|^2 \times \frac{1}{2} + |S_{25}|^2 \times \left(\bar{n}_{m,1} + \frac{1}{2}\right) + |S_{26}|^2 \times \left(\bar{n}_{m,2} + \frac{1}{2}\right) \\ &= \left(1 - \frac{1}{2\mathcal{C}}\right) \times \frac{1}{2} + 0 \times \frac{1}{2} + \frac{1}{4\mathcal{C}} \times \left(\bar{n}_{m,1} + \frac{1}{2}\right) + \frac{1}{4\mathcal{C}} \times \left(\bar{n}_{m,2} + \frac{1}{2}\right) \\ &= \frac{1}{2} + \frac{\bar{n}_{m,1} + \bar{n}_{m,2}}{4\mathcal{C}}, \end{aligned} \quad (25)$$

i.e. in the forward direction the noise contribution from the mechanical oscillators vanishes at large cooperativity  $\mathcal{C} \gg 1$ . Therefore, intriguingly, the noise emitted on resonance by the nonreciprocal device is not symmetric in the forward and backward directions.

### SUPPLEMENTARY NOTE 3: NOISE INTERFERENCE AS ORIGIN OF ASYMMETRIC NOISE EMISSION

In the previous note we concluded that the circuit emits more noise in the backward direction as compared to the forward direction. This is also corroborated by the experimental data, shown in Fig. 4 in the main text. In the following, in order to understand the different noise performance in the forward and backward direction, we consider two different points of view. First, we derive the scattering amplitude from one mechanical resonator to one cavity, eliminating the other two modes. In this picture, the imbalance can be understood as an interference of the two paths the noise can take in the circuit, analogously to the interference in the microwave transmission. Second, we eliminate the mechanical resonators, but taking their input noise into account. This leads to the same scattering matrix for the microwaves as discussed in the main text, but the mechanical noise appears as additional, effective noise input operators for the cavities. In the second formulation we can therefore use our knowledge of the microwave scattering matrix to deduce properties of the noise scattering.

Let us first consider the scattering from a mechanical resonator to cavities 1 and 2. Since in the experiment mechanical resonator 1 is strongly cross-damped due to off-resonant couplings, the noise emitted stems almost exclusively from resonator 2. If we are interested in the noise scattering from mechanical resonator 2 to cavity 2, we can eliminate the other two modes and drop their input noise. In frequency space, their equations of motion are

$$\begin{pmatrix} \chi_{c,1}^{-1}(\omega) & -ig_{11} \\ -ig_{11}^* & \chi_1^{-1}(\omega) \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{b}_1 \end{pmatrix} = \begin{pmatrix} ig_{12}\hat{b}_2 \\ ig_{21}^*\hat{a}_2 \end{pmatrix} + \text{noises}. \quad (26)$$

We drop the noise terms and solve for  $\hat{a}_1, \hat{b}_1$

$$\begin{aligned} \begin{pmatrix} \hat{a}_1(\omega) \\ \hat{b}_1(\omega) \end{pmatrix} &= \frac{1}{\chi_1^{-1}(\omega)\chi_{c,1}^{-1}(\omega) + |g_{11}|^2} \begin{pmatrix} \chi_1^{-1}(\omega) & ig_{11} \\ ig_{11}^* & \chi_{c,1}^{-1}(\omega) \end{pmatrix} \begin{pmatrix} ig_{12}\hat{b}_2(\omega) \\ ig_{21}^*\hat{a}_2(\omega) \end{pmatrix} \\ &\equiv \chi_{\hat{a}_1\hat{b}_1}(\omega) \begin{pmatrix} ig_{12} \\ ig_{21}^* \end{pmatrix} \begin{pmatrix} \hat{b}_2(\omega) \\ \hat{a}_2(\omega) \end{pmatrix}, \end{aligned} \quad (27)$$

where we defined the cavity susceptibility  $\chi_{c,i}^{-1}(\omega) = \kappa_i/2 - i\omega$  and the susceptibility of the coupled system of modes  $\hat{a}_1, \hat{b}_1$ ,  $\chi_{\hat{a}_1\hat{b}_1}(\omega)$ . We turn to the other two modes, the ones that we are actually interested in. For those, we have a similar equation, which can be obtained from interchanging  $1 \leftrightarrow 2$

$$\begin{pmatrix} \chi_2^{-1}(\omega) & -ig_{22}^* \\ -ig_{22} & \chi_{c,2}^{-1}(\omega) \end{pmatrix} \begin{pmatrix} \hat{b}_2(\omega) \\ \hat{a}_2(\omega) \end{pmatrix} = \begin{pmatrix} ig_{12}^* \\ ig_{21} \end{pmatrix} \begin{pmatrix} \hat{a}_1(\omega) \\ \hat{b}_1(\omega) \end{pmatrix} + \begin{pmatrix} \sqrt{\Gamma_{m,2}}\hat{b}_{2,\text{in}}(\omega) \\ \sqrt{\kappa_2}\hat{a}_{2,\text{in}}(\omega) \end{pmatrix}. \quad (28)$$

Eliminating the modes  $\hat{a}_1, \hat{b}_1$  with supplementary eq. (27), we arrive at

$$\begin{aligned} \begin{pmatrix} \sqrt{\Gamma_{m,2}}\hat{b}_{2,\text{in}}(\omega) \\ \sqrt{\kappa_2}\hat{a}_{2,\text{in}}(\omega) \end{pmatrix} &= \left[ \begin{pmatrix} \chi_2^{-1}(\omega) & -ig_{22}^* \\ -ig_{22} & \chi_{c,2}^{-1}(\omega) \end{pmatrix} - \frac{\begin{pmatrix} ig_{12}^* \\ ig_{21} \end{pmatrix} \begin{pmatrix} \chi_1^{-1}(\omega) & ig_{11} \\ ig_{11}^* & \chi_{c,1}^{-1}(\omega) \end{pmatrix} \begin{pmatrix} ig_{12} \\ ig_{21}^* \end{pmatrix}}{\chi_1^{-1}(\omega)\chi_{c,1}^{-1}(\omega) + |g_{11}|^2} \right] \begin{pmatrix} \hat{b}_2(\omega) \\ \hat{a}_2(\omega) \end{pmatrix} \\ &\equiv \left[ \chi_{\hat{b}_2\hat{a}_2}^{-1}(\omega) - \begin{pmatrix} ig_{12}^* \\ ig_{21} \end{pmatrix} \chi_{\hat{a}_1\hat{b}_1}(\omega) \begin{pmatrix} ig_{12} \\ ig_{21}^* \end{pmatrix} \right] \begin{pmatrix} \hat{b}_2(\omega) \\ \hat{a}_2(\omega) \end{pmatrix}. \end{aligned} \quad (29)$$

In the second line, we have formulated the equation in terms of the susceptibilities of the two subsystems  $(\hat{a}_1, \hat{b}_1)$  and  $(\hat{a}_2, \hat{b}_2)$ . This equation is a bit complicated, but we note that the coupling between  $\hat{a}_2$  and  $\hat{b}_2$  is

$$iT_{\hat{a}_2\hat{b}_2}(\omega) = -ig_{22} \left[ 1 - e^{-i\phi_p} \frac{\mathcal{C}_{12}\mathcal{C}_{21}/(\mathcal{C}_{22}\mathcal{C}_{11})}{1 + (\chi_{c,1}(\omega)\chi_1(\omega)|g_{11}|^2)^{-1}} \right]. \quad (30)$$

Analogously, changing the indices referring to the cavity, we obtain the coupling between  $\hat{a}_1$  and  $\hat{b}_2$

$$iT_{\hat{a}_1\hat{b}_2}(\omega) = -ig_{12} \left[ 1 - e^{+i\phi_p} \frac{\mathcal{C}_{11}\mathcal{C}_{22}/(\mathcal{C}_{12}\mathcal{C}_{21})}{1 + (\chi_{c,2}(\omega)\chi_1(\omega)|g_{21}|^2)^{-1}} \right], \quad (31)$$

The coupling phase  $\phi_p$  appears as the relative phase between indirect and direct coupling path, as for the microwave signal transmission. Equations (30) and (31) demonstrate that the transmission of noise from the mechanical resonators to the microwave cavities is subject to interference, which ultimately leads to the difference in noise emitted in the forward versus the backward direction.

In a second picture, we can also understand the mechanical noise interference in terms of the nonreciprocity in the scattering matrix for the microwave modes. In order to do so, we solve the equations of motion for the mechanical resonators (given in supplementary eq. (6)), which leads to

$$\hat{b}_j(\omega) = \chi_j(\omega) \left[ i \sum_i g_{ij}^* \hat{a}_i(\omega) + \sqrt{\Gamma_{m,j}} \hat{b}_{j,\text{in}}(\omega) \right]. \quad (32)$$

We obtain equations that only relate the cavities

$$\begin{pmatrix} \chi_{c,1}^{-1}(\omega) + iT_{11}(\omega) & iT_{12}(\omega) \\ iT_{21}(\omega) & \chi_{c,2}^{-1}(\omega) + iT_{22}(\omega) \end{pmatrix} \begin{pmatrix} \hat{a}_1(\omega) \\ \hat{a}_2(\omega) \end{pmatrix} = i \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} \sqrt{\Gamma_{m,1}} \chi_1(\omega) \hat{b}_{1,\text{in}}(\omega) \\ \sqrt{\Gamma_{m,2}} \chi_2(\omega) \hat{b}_{2,\text{in}}(\omega) \end{pmatrix} + \begin{pmatrix} \sqrt{\kappa_1} \hat{a}_{1,\text{in}}(\omega) \\ \sqrt{\kappa_2} \hat{a}_{2,\text{in}}(\omega) \end{pmatrix}, \quad (33)$$

where

$$iT_{ij}(\omega) \equiv -i \sum_k \chi_k(\omega) g_{ik} g_{jk}^*. \quad (34)$$

We can think of mechanical noise as coloured and correlated noise in the optical inputs. That is, consider the replacement

$$\begin{pmatrix} \sqrt{\kappa_1} \hat{c}_{1,\text{in}}(\omega) \\ \sqrt{\kappa_2} \hat{c}_{2,\text{in}}(\omega) \end{pmatrix} \equiv i \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} \sqrt{\Gamma_{m,1}} \chi_1(\omega) \hat{b}_{1,\text{in}}(\omega) \\ \sqrt{\Gamma_{m,2}} \chi_2(\omega) \hat{b}_{2,\text{in}}(\omega) \end{pmatrix}. \quad (35)$$

The effective noise  $\hat{c}_{i,\text{in}}$  is both coloured  $\langle \hat{c}_{1,\text{in}}^\dagger(\omega) \hat{c}_{1,\text{in}}(\omega') \rangle \neq \delta(\omega + \omega') \bar{n}_{1,\text{eff}}$  and correlated  $\langle \hat{c}_{1,\text{in}}^\dagger(\omega) \hat{c}_{2,\text{in}}(\omega') \rangle \neq 0$ .

Using the input-output relation  $\hat{a}_{\text{out}} = \hat{a}_{\text{in}} - \sqrt{\kappa} \hat{a}$ , the cavity output is given by

$$\begin{pmatrix} \hat{a}_{1,\text{out}}(\omega) \\ \hat{a}_{2,\text{out}}(\omega) \end{pmatrix} = S(\omega) \begin{pmatrix} \hat{a}_{1,\text{in}}(\omega) \\ \hat{a}_{2,\text{in}}(\omega) \end{pmatrix} + [S(\omega) - \mathbb{1}_2] \begin{pmatrix} \hat{c}_{1,\text{in}}(\omega) \\ \hat{c}_{2,\text{in}}(\omega) \end{pmatrix}, \quad (36)$$

where in the last step we have identified the 2-by-2 optical scattering matrix  $S(\omega)$  that relates the cavity inputs to the outputs  $\hat{a}_{i,\text{out}}(\omega) = \sum_j S_{ij}(\omega) \hat{a}_{j,\text{in}}(\omega)$ . The fact that supplementary eq. (36) contains mechanical noise as well, but can be written entirely in terms of the optical scattering matrix constitutes the central result here. Since the two effective input noises  $\hat{c}_{i,\text{in}}$  are coloured and correlated, they can interfere.

Most importantly, we can consider what happens when the circuit is impedance matched to the signal and perfectly isolating. We choose the detunings  $\delta_1 = \Gamma_{m,1} \delta/2$ ,  $\delta_2 = -\Gamma_{m,2} \delta/2$ , for some dimensionless parameter  $\delta$ . For simplicity, let us choose all cooperativities to be equal  $\mathcal{C} = \mathcal{C}_{ij}$ . For  $\delta^2 = 2\mathcal{C} - 1$  (impedance matching), the optical scattering matrix of the isolator is (up to some irrelevant phase)

$$S(0) = \begin{pmatrix} 0 & 0 \\ \sqrt{1 - 1/(2\mathcal{C})} & 0 \end{pmatrix} \equiv T \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (37)$$

The cavity output on resonance is

$$\begin{pmatrix} \hat{a}_{1,\text{out}} \\ \hat{a}_{2,\text{out}} \end{pmatrix} = T \begin{pmatrix} 0 \\ \hat{a}_{1,\text{in}} \end{pmatrix} - \frac{i}{\sqrt{2}} \mathcal{C} \begin{pmatrix} e^{i\phi_p} & 1 \\ 1 - T e^{i\phi_p} & 1 - T \end{pmatrix} \begin{pmatrix} \hat{b}_{1,\text{in}}(0) \\ \hat{b}_{2,\text{in}}(0) \end{pmatrix}. \quad (38)$$

As  $\mathcal{C} \rightarrow \infty$ ,  $T \rightarrow 1$  and  $\phi_p = \arccos(1 - 1/\mathcal{C}) \rightarrow 0$ , such that the second cavity does not receive any noise, which is due to an interference of  $\hat{c}_{1,\text{in}}$  with  $\hat{c}_{2,\text{in}}$ . In the backward direction, no interference can take place, since cavity 2 is isolated from cavity 1. As a consequence, the number of noise quanta emerging from cavity 1 on resonance is  $N_{\text{bw}} = (\bar{n}_{m,1} + \bar{n}_{m,2} + 1)/2$ .

**SUPPLEMENTARY NOTE 4: OPTOMECHANICAL CIRCULATOR**

In this note we present a scheme for a circulator based on the same principles as the isolator previously discussed. The scheme naturally overcomes the shortcomings of the isolator, becoming wideband and quantum limited in the high cooperativity limit.

We consider three microwave modes (described by their annihilation operators  $\hat{a}_1, \hat{a}_2, \hat{a}_3$ ) with resonance frequencies  $\omega_{c,1}, \omega_{c,2}, \omega_{c,3}$  and dissipation rates  $\kappa_1, \kappa_2, \kappa_3$ . These three microwave modes are coupled to two mechanical modes (described by the annihilation operators  $\hat{b}_1, \hat{b}_2$ ) with resonance frequencies  $\Omega_1, \Omega_2$  and dissipation rates  $\Gamma_{m,1}$  and  $\Gamma_{m,2}$ . The optomechanical coupling strengths  $g_{ij}$  are taken to be real and we define three phases  $\phi_1, \phi_2$  and  $\phi_3$  associated respectively to the couplings  $g_{11}, g_{21}$  and  $g_{31}$ . The three cavities are driven with two microwave tones each. These six tones are close to the lower motional sidebands, with detunings of  $\Delta_{11} = \Delta_{21} = \Delta_{31} = -\Omega_1 + \delta_1$  and  $\Delta_{12} = \Delta_{22} = \Delta_{32} = -\Omega_2 + \delta_2$ . The cooperativities are set to be equal for all couplings, with  $\mathcal{C} = \mathcal{C}_{ij} = \frac{4g_{ij}}{\kappa_i\Gamma_j}$ .

The linearised Hamiltonian that describes the system, in a frame rotating with the cavity frequencies and keeping only time-constant terms is given by

$$\begin{aligned} \hat{H} = & \delta_1 \hat{b}_1^\dagger \hat{b}_1 + \delta_2 \hat{b}_2^\dagger \hat{b}_2 \\ & + g_{11} (\hat{a}_1^\dagger \hat{b}_1 e^{i\phi_1} + \hat{a}_1 \hat{b}_1^\dagger e^{-i\phi_1}) + g_{12} (\hat{a}_1^\dagger \hat{b}_2 + \hat{a}_1 \hat{b}_2^\dagger) \\ & + g_{21} (\hat{a}_2^\dagger \hat{b}_1 e^{i\phi_2} + \hat{a}_2 \hat{b}_1^\dagger e^{-i\phi_2}) + g_{22} (\hat{a}_2^\dagger \hat{b}_2 + \hat{a}_2 \hat{b}_2^\dagger) \\ & + g_{31} (\hat{a}_3^\dagger \hat{b}_1 e^{i\phi_3} + \hat{a}_3 \hat{b}_1^\dagger e^{-i\phi_3}) + g_{32} (\hat{a}_3^\dagger \hat{b}_2 + \hat{a}_3 \hat{b}_2^\dagger). \end{aligned} \quad (39)$$

From this Hamiltonian, we derive the equations of motion for our system in the matrix form (as in supplementary eq. (6)) with  $\mathbf{u} = (\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{b}_1, \hat{b}_2)^T$ ,  $\mathbf{u}_{\text{in}} = (\hat{a}_{1,\text{in}}, \hat{a}_{2,\text{in}}, \hat{a}_{3,\text{in}}, \hat{a}_{1,\text{in}}^{(0)}, \hat{a}_{2,\text{in}}^{(0)}, \hat{a}_{3,\text{in}}^{(0)}, \hat{b}_{1,\text{in}}, \hat{b}_{2,\text{in}})^T$  and  $\mathbf{u}_{\text{out}} = (\hat{a}_{1,\text{out}}, \hat{a}_{2,\text{out}}, \hat{a}_{3,\text{out}}, \hat{a}_{1,\text{out}}^{(0)}, \hat{a}_{2,\text{out}}^{(0)}, \hat{a}_{3,\text{out}}^{(0)}, \hat{b}_{1,\text{out}}, \hat{b}_{2,\text{out}})^T$ . The matrix  $M$  is here given by

$$M = \begin{pmatrix} -\frac{\kappa_1}{2} & 0 & 0 & -ig_{11}e^{i\phi_1} & -ig_{12} \\ 0 & -\frac{\kappa_2}{2} & 0 & -ig_{21}e^{i\phi_2} & -ig_{22} \\ 0 & 0 & -\frac{\kappa_3}{2} & -ig_{31}e^{i\phi_3} & -ig_{32} \\ -ig_{11}e^{-i\phi_1} & -ig_{21}e^{-i\phi_2} & -ig_{31}e^{-i\phi_3} & -\frac{\Gamma_{m,1}}{2} + i\delta_1 & 0 \\ -ig_{12} & -ig_{22} & -ig_{32} & 0 & -\frac{\Gamma_{m,2}}{2} + i\delta_2 \end{pmatrix}, \quad (40)$$

where the cavity dissipation rates are the sum of the external and internal dissipation rates, i.e.  $\kappa_i = \kappa_{\text{ex},i} + \kappa_{0,i}$ . The matrix  $L$  is here

$$L = \begin{pmatrix} \sqrt{\kappa_{\text{ex},1}} & 0 & 0 & \sqrt{\kappa_{0,1}} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\kappa_{\text{ex},2}} & 0 & 0 & \sqrt{\kappa_{0,2}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\kappa_{\text{ex},3}} & 0 & 0 & \sqrt{\kappa_{0,3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\Gamma_{m,1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\Gamma_{m,2}} \end{pmatrix}. \quad (41)$$

Using the input-output relation (supplementary eq. (9)) and the matrix form of the equations of motion (supplementary eq. (6)), we can compute the scattering matrix  $S(\omega)$  similarly to supplementary eq. (11).

We choose to operate the circulator in a way that suppresses the propagation in the clockwise direction, i.e.  $S_{12}(0) = 0$ ,  $S_{23}(0) = 0$ , and  $S_{31}(0) = 0$ . For this suppression to occur on resonance ( $\omega = 0$ ),  $\delta_1$  must scale with  $\Gamma_{m,1}$  and  $\delta_2$  with  $\Gamma_{m,2}$  so we define  $\delta_1 = \alpha\Gamma_{m,1}$  and  $\delta_2 = \beta\Gamma_{m,2}$ . The equations corresponding to  $S_{12}(0) = S_{23}(0) = S_{31}(0) = 0$  are

$$-2i\alpha - 2i\beta e^{i(\phi_1 - \phi_2)} - \mathcal{C}(1 - e^{i(\phi_1 - \phi_3)} - e^{i(\phi_3 - \phi_2)} + e^{i(\phi_1 - \phi_2)}) - (1 + e^{i(\phi_1 - \phi_2)}) = 0, \quad (42)$$

$$-2i\alpha - 2i\beta e^{i(\phi_2 - \phi_3)} - \mathcal{C}(1 - e^{i(\phi_2 - \phi_1)} - e^{i(\phi_1 - \phi_3)} + e^{i(\phi_2 - \phi_3)}) - (1 + e^{i(\phi_2 - \phi_3)}) = 0, \quad (43)$$

$$-2i\alpha - 2i\beta e^{i(\phi_3 - \phi_1)} - \mathcal{C}(1 - e^{i(\phi_3 - \phi_2)} - e^{i(\phi_2 - \phi_1)} + e^{i(\phi_3 - \phi_1)}) - (1 + e^{i(\phi_3 - \phi_1)}) = 0. \quad (44)$$

Analysing this set of equations, we see that only two phases are independent. Setting  $\phi_1 = 2\pi/3$ ,  $\phi_2 = -2\pi/3$  and  $\phi_3 = 0$  leads to a set of fully degenerate equations. We then obtain  $S_{13}(0) = S_{32}(0) = S_{21}(0) = 0$  if

$$2\sqrt{3}\beta - 3\mathcal{C} - 1 + i(4\alpha - 2\beta + 3\sqrt{3}\mathcal{C} + \sqrt{3}) = 0. \quad (45)$$

Solving supplementary eq. (45) with respect to the cooperativity  $\mathcal{C}$  gives

$$\mathcal{C} = \frac{2\beta}{\sqrt{3}} - \frac{1}{3} \quad \text{and} \quad \alpha = -\beta. \quad (46)$$

We note that if  $\alpha \neq -\beta$ , then  $\mathcal{C}$  must contain an imaginary part leading to complex coupling strengths, which is inconsistent with their definition as being real. Moreover,  $\mathcal{C}$  must be positive (such that  $g_{ij} \in \mathbb{R}$ ) and non-zero (else  $g_{ij} = 0$ ). The lower bound for  $\beta$  is thus given by  $1/(2\sqrt{3})$ .

We can write the transmission in the counter-clockwise direction ( $|S_{13}|^2$ ,  $|S_{32}|^2$  and  $|S_{21}|^2$ ) on resonance as

$$|S_{13}|^2 = \frac{\kappa_{\text{ex},1}\kappa_{\text{ex},3}}{\kappa_1\kappa_3} \frac{1}{(1 + \frac{1}{3\mathcal{C}})^2}, \quad (47)$$

$$|S_{32}|^2 = \frac{\kappa_{\text{ex},3}\kappa_{\text{ex},2}}{\kappa_3\kappa_2} \frac{1}{(1 + \frac{1}{3\mathcal{C}})^2}, \quad (48)$$

$$|S_{21}|^2 = \frac{\kappa_{\text{ex},2}\kappa_{\text{ex},1}}{\kappa_2\kappa_1} \frac{1}{(1 + \frac{1}{3\mathcal{C}})^2}. \quad (49)$$

We find that, in the case of overcoupled cavities  $\kappa_i \approx \kappa_{\text{ex},i}$ , the transmission approaches unity with increasing cooperativity.

The symmetrised output noise spectra is computed as in the supplementary note 2. In the limit of overcoupled cavities,  $\kappa_i \approx \kappa_{\text{ex},i}$ , the noise emitted on resonance at each port is given by

$$N = \frac{1}{2} + \frac{3\mathcal{C}}{(3\mathcal{C} + 1)^2} (\bar{n}_{\text{m},1} + \bar{n}_{\text{m},2}). \quad (50)$$

In the limit of large cooperativity, the noise contribution from the mechanical oscillators is entirely suppressed, leaving only vacuum noise amounting to half a quantum.

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